MA267-10 Groups and Rings

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Nicholas Jackson
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This is an introductory abstract algebra module. As the title suggests, the two main objects of study are groups and rings.

Module aims

This is a standard first abstract algebra module, roughly based on the current version of Algebra-2: Groups and Rings. It consists of 5 weeks of Group Theory and 5 weeks of Ring Theory. Some of the heavier ring-theoretic topics in the current Algebra-2 are dropped. The module gives access to all Algebra options in year 2 and 3 as well as Number Theory in year 2. I am referring to chapters in the 2019 lecture notes.

The students have done some algebra in Sets and Numbers in year 1. The proficiency with operations in the symmetric group is assumed. For instance, the students may have seen the following notions: groups (abelian, cyclic), order of an element, ring, field. On the other hand, no proficiency in these notions is assumed.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Group Theory: motivating examples (numbers, dihedral group, quaternionic group, matrix groups), elementary properties, subgroup, coset, Lagrange's theorem, quotient group, isomorphism theorem, free group, group given by generators and relations, group action, G-set G/H, orbit, stabiliser, the orbit-stabiliser theorem, conjugacy class, classes in S_n, classification of groups up to order 8.
- Ring Theory: commutative and non-commutative ring, domain, examples (Z[x], Z/nZ, F[x], F[x]/(f)), ideal, quotient ring, isomorphism theorem, Chinese remainder theorem for Z/nZ and F[x]/(f), unit group, prime and irreducible element, factorization, Euclidean domain, characteristic of a field, unique factorization domain, ED is UFD, Eisenstein criterion, Gauss lemma, cyclotomic polynomial, finite subgroups of units in fields.
- Module Theory: module, free module, internal and external direct sum, free abelian group, unimodular Smith normal form, the fundamental theorem of finitely generated abelian groups.
- List of covered algebraic definitions: group or ring homomorphism (including kernel, image, isomorphism), direct product, coset, normal subgroup, quotient group, ideal, quotient ring, domain, irreducible element, prime element, euclidean domain, unique factorisation domain, direct product, free group, generators and relations, module, free module, direct sum, unimodular Smith normal form, action, orbit, stabiliser.

Learning outcomes

By the end of the module, students should be able to:

- have a working knowledge of the main constructions and concepts of theories of groups and rings
- recognise, classify and construct examples of groups and rings with specified properties by applying the algebraic concepts
- get the working knowledge of the understand the definition of various types of ring, and be familiar with a number of examples, including numbers, polynomials and Z/nZ

Indicative reading list

Samir Siksek, Introduction to Abstract Algebra lecture notes,
Ronald Solomon, Abstract Algebra, Brooks/Cole, 2003.
Niels Lauritzen, Concrete Abstract Algebra, Cambridge University Press, 2003
John B. Fraleigh, A first course in abstract algebra, Pearson, 2002
Joseph A. Gallian, Contemporary Abstract Algebra, Cengage Learning, 2012

View reading list on Talis Aspire

Subject specific skills

Students will improve their skills in thinking algebraically in a variety of settings. This includes working with axiomatic definitions of algebraic objects and analysing the structure and relationships between algebraic objects using fundamental tools such as subobjects and homomorphisms, laying a foundation for future study in algebra, number theory and algebraic geometry.

Transferable skills

The module emphasises the power of generalisation and abstraction. Students will improve their ability to analyse abstract concepts and to solve problems by selecting and applying appropriate abstract tools.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (20%)
Online learning (independent)	9 sessions of 1 hour (9%)
Private study	13 hours (13%)
Assessment	58 hours (58%)
Total	100 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments	15%	20 hours
Examination	85%	38 hours

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes. Exam feedback is given.

Past exam papers for MA267

Availability

Courses

This module is Core for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 2 of GG14 Mathematics and Statistics
  - Year 2 of GG14 Mathematics and Statistics

This module is Option list B for:

- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics