MA260-10 Norms, Metrics and Topologies

23/24

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 2

Module leader
: Richard Sharp

Credit value
: 10

Module duration
: 10 weeks

Assessment
: Multiple

Study locations
: University of Warwick main campus, Coventry Primary
: Distance or Online Delivery

Description

Introductory description

This is a module bridging Y1 Analysis and Y2 Analysis modules. The concepts of convergence, continuity, convergence and compactness are studied in the more general setting. This enables development of multi-dimensional and infinite-dimensional Analysis in consequent modules.

[Module web page]

Module aims

To introduce the notions of Normed Space, Metric Space and Topological Space, and the fundamental properties of Compactness, Connectedness and Completeness that they may possess.

Overall, this is an Analysis module, not a Topology module. The notion of topology is introduced but the focus is on the topologies, naturally occurring in Analysis. There will be no emphasis on topological spaces.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Recap of Y1 material: properties of intervals in R and basic geometry in R^n
- Normed spaces: definitions, norms on R^n, spaces of linear operators, spaces of functions
- Metric spaces: norms as metrics, metric on subsets, open and closed sets, convergence, continuity, uniform convergence of functions and applications (interchange of limits)
- Topological spaces: basis, sub-basis, closure, interior, boundary, product topology, Hausdorff property, continuity, homeomorphisms and topological properties, topologically equivalent metrics
- Connectedness: unions and products of connected sets, components, path-connected spaces, connected subsets of R and R^n
- Compactness: Heine-Borel Theorem, equivalence of all norms on R^n, continuous functions on compact sets (Extreme Value Theorem and uniform continuity), sequential compactness of metric spaces
- Completeness: R^n is complete, completion, Contraction Mapping Theorem, Arzela-Ascoli Theorem, applications to existence of solutions of ODEs

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate understanding of the basic concepts, theorems and calculations of Normed, Metric and Topological Spaces.
- Demonstrate understanding of the open-set definition of continuity and its relation to previous notions of continuity, and applications to open or closed sets.
- Demonstrate understanding of the basic concepts, theorems and calculations of the concepts of Compactness, Connectedness and Completeness (CCC).
- Demonstrate understanding of the connections that arise between CCC, their relations under continuous maps, and simple applications.

Indicative reading list

1. W A Sutherland, Introduction to Metric and Topological Spaces, OUP.
2. E T Copson, Metric Spaces, CUP.
3. G W Simmons, Introduction to Topology and Modern Analysis, McGraw Hill. (More advanced, although it starts at the beginning; helpful for several third year and MMath modules in analysis).

Subject specific skills

Familiarity with different ways of formulating convergence and continuity, and the relationship between them. Ability to use compactness and completeness arguments as part of larger proofs, frequently required in mathematical applications.

Transferable skills

Analytical and problem-solving skills as for any module in abstract mathematics. Facility for independent study and self motivation.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (20%)
Seminars	5 sessions of 1 hour (5%)
Other activity	10 hours (10%)
Private study	25 hours (25%)
Assessment	40 hours (40%)
Total	100 hours

Private study description

self-working: reviewing lectured material and accompanying supplementary materials; working on both summative and formative coursework; revising for exams.

Other activity description

Collaborative project

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Problem sheets	15%	16 hours
2 hour examination (Summer)	85%	24 hours
2 hour examination - no books allowed		

	Weighting	Study time

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked homework (formative) is returned and discussed in smaller classes and exam feedback.

Past exam papers for MA260

Availability

Courses

This module is Core for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year