MA222-10 Metric Spaces

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Andras Mathe
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This is a module bridging Y1 Analysis and Y2 Analysis modules. The concepts of convergence, continuity, convergence and compactness are studied in the more general setting. This enables development of multi-dimensional and infinite-dimensional Analysis in consequent modules.

Module web page

Module aims

The module introduces the notions of normed space, metric space and topological space. In this setting we introduce open sets and closed sets, and discuss their relationship to convergence and continuity. We will introduce the concepts of compactness and connectedness, how they relate to continuity. We also consider completeness. This material provides the bridge from analysis on the real line, as considered in earlier analysis modules, to a much more flexible and general framework. This is essential for many later modules in Years 3 and 4, particular pure mathematics modules involving analysis, geometry or topology.

Overall, this is an Analysis module, not a Topology module. The notion of topology is introduced but the focus is on the topologies, naturally occurring in Analysis. There will be no emphasis on topological spaces.

As the students are coming from different first year backgrounds, a special care will be taken at

the start to build on the familiar concepts from the first year Analysis and Linear Algebra. Metric Spaces will be taught slightly differently with greater emphasis on examples.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Recap of Y1 material: properties of intervals in R and basic geometry in R^n
- Normed spaces: definitions, norms on R^n, spaces of linear operators, spaces of functions
- Metric spaces: norms as metrics, metric on subsets, open and closed sets, convergence, continuity, uniform convergence of functions and applications (interchange of limits)
- Topological spaces: basis, sub-basis, closure, interior, boundary, product topology, Hausdorff property, continuity, homeomorphisms and topological properties, topologically equivalent metrics
- Connectedness: unions and products of connected sets, components, path-connected spaces, connected subsets of R and R^n
- Compactness: Heine-Borel Theorem, equivalence of all norms on R^n, continuous functions on compact sets (Extreme Value Theorem and uniform continuity), sequential compactness of metric spaces
- Completeness: R^n is complete, completion, Contraction Mapping Theorem, Arzela-Ascoli Theorem, applications to existence of solutions of ODEs

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate understanding of the basic concepts, theorems and calculations of Normed, Metric and Topological Spaces.
- Demonstrate understanding of the open-set definition of continuity and its relation to previous notions of continuity, and applications to open or closed sets.
- Demonstrate understanding of the basic concepts, theorems and calculations of the concepts of Compactness, Connectedness and Completeness (CCC).
- Demonstrate understanding of the connections that arise between CCC, their relations under continuous maps, and simple applications.

Indicative reading list

1. W A Sutherland, Introduction to Metric and Topological Spaces, OUP.
2. E T Copson, Metric Spaces, CUP.
3. W Rudin, Principles of Mathematical Analysis, McGraw Hill.
4. G W Simmons, Introduction to Topology and Modern Analysis, McGraw Hill. (More advanced, although it starts at the beginning; helpful for several third year and MMath modules in analysis).
5. A M Gleason, Fundamentals of Abstract Analysis, Jones and Bartlett.

Subject specific skills

Students will develop understanding of metric and topological spaces, and of convergence and continuity in these settings. They will be able to characterise convergence and continuity in terms of open sets. They will have a good knowledge of a variety of examples of metric spaces and be able to determine whether or not they are topologically equivalent. They will be familiar with topological spaces as a generalisation of metric spaces and understand the role of metrisability. They will understand the concept of compactness and how it is related to continuity. They will become familiar with the notion of connectedness and understand non-trivial examples. They will understand Cauchy sequences and completeness of metric spaces and understand how to construct the completion of a non-complete space.

Transferable skills

The module deals with abstract mathematical concepts, where examples may be removed from students' normal intuition. Therefore, students who have successfully completed the module will have developed their skills in reasoning in an abstract setting, for example from a set of axioms. They will have the background to carry of further study in the area, and to apply the techniques they have learned to various areas of analysis, geometry and topology. More generally, they will have had the opportunity to develop their analytic skills through the study of complex and abstract systems.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (9%)
Private study	71 hours (71%)
Total	100 hours

Private study description

71 hours to review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments	15%	
Examination	85%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA222

Availability

Courses

This module is Core for:

- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 2 of GG14 Mathematics and Statistics
  - Year 2 of GG14 Mathematics and Statistics

This module is Option list A for:

- USTA-G302 Undergraduate Data Science
  - Year 2 of G302 Data Science
  - Year 2 of G302 Data Science

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics